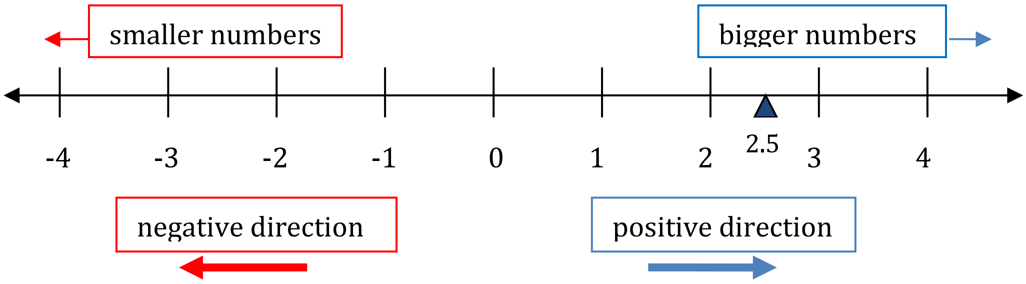
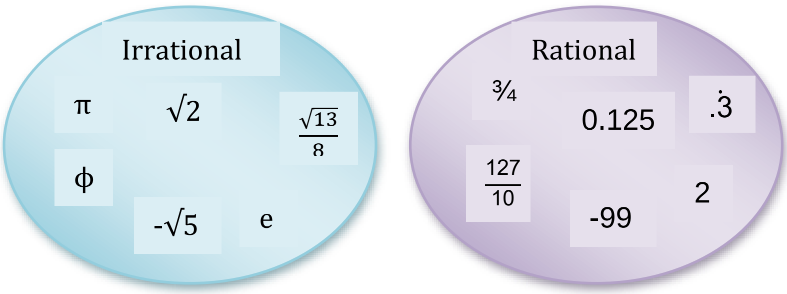
Module 1: Numbers

Real numbers can be represented as points on a real **number line**.



Every real number is a distance from the point 0 on the number line and its sign (negative or positive) indicates the direction along the line. So -5 is 5 units from 0 in the negative direction, which is in the opposite direction to +5. A negative sign is used to indicate a change to the **opposite** direction, so -(-5) is + 5.

KINDS OF NUMBERS



{1, 2 , 3 , 4 , 5 , 6 , 7, …} is the set of **natural** (counting) numbers, whereas

{0 , 1 , 2 , 3 , 4 , 5 , 6 , 7, …} is the set of **whole** numbers, zero is included. In both positive and negative directions there are **integers**:{ ... -4 , -3 , -2 , -1 , 0 , 1, 2 , 3 , 4 , 5 ...}

Rational numbers can be written as the quotient of two integers, with a non-zero denominator, e.g. $^2/_3 = 0.6666\ldots$, $-1^1/_4 = -\frac{5}{4}$ and $15.3 = \frac{153}{10}$. Rational numbers can be expressed as a fraction or as a terminating or repeating decimal.

Irrational numbers are all the numbers on the real number line that are not rational, e.g. $\sqrt{2}$, $-\sqrt{10}$, π , $^{\sqrt{5}}/_2$. They cannot be expressed as a fraction, the decimal neither terminates or is repeating.

The **absolute value** $|x|$ of a number x is its distance from 0 along the real number line (ignoring its sign). The absolute value is always positive, $|-4| = 4$

A **prime** number is a natural number that has exactly two factors, 1 and itself. The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

A **composite** number is a natural number that has more than two factors. For example, each of the natural numbers 1, 2, 3, 4, 6, and 12 divide exactly into 12, therefore 12 is a composite number with 6 factors.

Exercise 1

List all the numbers in the following set,

{ -9.001, $-\sqrt{7}$, 10 , $-1\frac{1}{4}$, 0 , $\sqrt{5}$, 3 , 8.94 , $-\frac{3}{5}$, 7, -2 , 21 , 37 }, that are:

(a) natural (b) whole (c) integers (d) negative real numbers

(e) rational (f) irrational (g) prime (h) composite

POWERS

A **square number** is the number obtained by multiplying a whole number by itself, for example 1x1=1, 2x2=4, 3x3=9 and 25x25=625. The **square root** of 625 is a number, which multiplied by itself gives 625, so the square root of 625 is 25.

The **mathematical notation** for writing 2x2 is 2^2 and the symbol for taking the square root of a number is $\sqrt{\ }$. For example, $11^2 = 121$ and $\sqrt{121} = 11$. Your **scientific calculator** may have separate buttons for the square and square root functions, or these two functions may be located at one button and the $[shift]$ button must be used first to activate the $\sqrt{\ }$ function.

Similarly, a **cubed number** is a number multiplied by itself 3 times, for example 2 x 2 x 2 = 2^3 = 8 and the **cube root** of 8 is 2. The mathematical notation for cube root is $\sqrt[3]{\ }$, so for example if $4^3 = 64$ then $\sqrt[3]{64} = 4$

In general, a number may be multiplied by itself any number of times, for example, 6 x 6 x 6 x 6 = 6^4 = 1296. '6 to the power of 4 equals 1296' is the same as multiplying 6 by itself 4 times. The **power** denotes the number of times to multiple the given number by itself. For example, 4^5 = 1024 means multiply 4 by itself 5 times. 4 x 4 x 4 x 4 x 4 = 1024.

On your **scientific calculator**, the general power button is usually [y^x] or [^], and the general root button is $[\sqrt[y]{x}]$ or $[\sqrt[x]{\ }]$.

Use your <u>**calculator**</u> to check the answers in the following examples.

$8^2 = 64$ $\sqrt{529} = 23$

$11^3 = 1331$ $\sqrt[3]{343} = 7$

$6^5 = 7776$ $\sqrt[5]{7776} = 6$

$1.853^2 = 3.433609$ $\sqrt{0.7396} = 0.86$

INEQUALITIES

Symbol	Meaning	Examples	Meaning
>	Greater than	$\sqrt{34} > 5$	$\sqrt{34}$ is greater than 5
<	Less than	-21.5 < -21.4	-21.5 is less than -21.4

Inequalities can also be used to show a quantity lies between two numbers. For example, $5 < \sqrt{34}$. It is also true that $\sqrt{34} < 6$, so it follows that $\sqrt{34}$ is between 5 and 6. This statement can be written as $5 < \sqrt{34} < 6$.

Symbol	Meaning
≥	Greater than or equal to
≤	Less than or equal to

Examples	Meaning
$x \geq 2$	Is the set of all numbers greater than or equal to 2.
$x \leq 0$	Is the set of all numbers less than or equal to 0.
$2 \leq x \leq 5$	The set of all numbers between 2 and 5, including 2 and 5.
$-3 < x < 0$	The set of all numbers between -3 and 0.
$-4 \leq x < -1$	The set of all numbers between -4 and -1, including -4.
$-2 < x \leq 10$	The set of all numbers between -2 and 10, including 10.

Exercise 2

Which of the following are true or false.

(a) $-1\frac{1}{4} < -1$ (b) $1 > -\frac{1}{2}$ (c) $|2| = 2$

(d) $|-6| = 6$ (e) $(-2)^3 = 8$ (f) $\sqrt{97} > 10$

(g) $-2.01 > -2$ (h) $|-5| > |3|$ (i) $23 < \sqrt{577} < 24$

Match each word statement below, with the inequality statements. (x represents the numbers which satisfies the given condition.)

(j) The set of all real numbers between -5 and 4.	$x \leq 10$
(k) All real numbers which are at least 10.	$-5 < x < 4$
(l) The set of all real numbers between -5 and 4, inclusive of the end points.	$-5 < x \leq 4$
(m) All real numbers which do not exceed 10	$x \geq 10$
(n) The set of real numbers more than -5 but less than or equal to 4.	$-5 \leq x \leq 4$

FRACTIONS

Fractions are used to represent parts of whole numbers and are in the form $\frac{\text{numerator}}{\text{denominator}}$

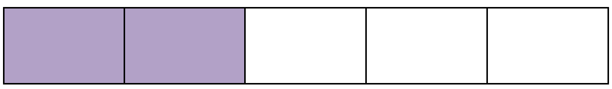
These must both be whole numbers; the denominator cannot be zero.

The denominator represents how many parts the whole has been divided into, and the numerator indicates the number of those parts being considered. Any whole number can be written as a fraction with a denominator of 1, so $4 = \frac{4}{1}$

Examples

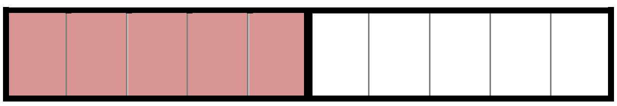
$\frac{1}{5}$ is one part of a whole divided into 5 equal parts:

$\frac{2}{5}$ is two parts of a whole divided into 5 equal parts:



$\frac{1}{2} = \frac{5}{10}$ as each equal part can be split further into 5 equal pieces.

So $\frac{1}{2} = \frac{1\times5}{2\times5} = \frac{5}{10}$. Its simplest form is $\frac{1}{2}$:



Proper fractions are less than a whole unit having a smaller numerator than denominator, e.g. $^2/_5$.

Improper fractions are more than a whole unit with a larger numerator than denominator, e.g. $^{11}/_5$

Mixed numbers consist of a whole part and a fraction, e.g. $3\ ^4/_5$.

Equivalent fractions have the same simplest form, e.g. $^4/_8\ =\ ^3/_6$ as both can be simplified to $^1/_2$.

We can easily convert between these types of fractions. Remember a fraction line is a division line.

So $\frac{11}{5} = \frac{10}{5} + \frac{1}{5} = 2 + \frac{1}{5} = 2\frac{1}{5}$ and $3\frac{4}{5} = \frac{3}{1} + \frac{4}{5} = \left[\frac{3\times 5}{1\times 5}\right] + \frac{4}{5} = \frac{15}{5} + \frac{4}{5} = \frac{19}{5}$

(5 parts make a whole) (3 wholes split into 5 parts each)

Exercise 3

Complete the following: (a) $\frac{4}{5} = \frac{\quad}{10}$ (b) $\frac{3}{4} = \frac{\quad}{20}$ (c) $\frac{1}{3} = \frac{25}{\quad}$ (d) $\frac{3}{8} = \frac{30}{\quad}$

Exercise 4

Simplify (a) $\frac{12}{16} =$ (b) $\frac{25}{40} =$ (c) $\frac{21}{39} =$ (d) $\frac{35}{49} =$ (e) $\frac{22}{33} =$

Exercise 5

Convert these mixed numbers to improper fractions; and improper fractions to mixed numbers.

(a) $1\frac{2}{3} =$ (b) $2\frac{1}{2} =$ (c) $9\frac{2}{7} =$ (d) $\frac{17}{5} =$ (e) $\frac{50}{4}$ (f) $\frac{22}{3} =$

Comparing, Adding and Subtracting Fractions

We must first rewrite each fraction as an equivalent fraction, with the same denominator. Then each part of both fractions will be of the same size and the fractions can be compared, or added or subtracted.

For example, to compare $^4/_5$ and $^2/_3$ we change both to fractions with a denominator of 15, then we can see

that $\frac{4}{5} = \frac{12}{15}$ *is larger than* $\frac{2}{3} = \frac{10}{15}$ *by* $\frac{2}{15}$.

Adding these gives $\frac{4}{5} + \frac{2}{3} = \frac{12}{15} + \frac{10}{15} = \frac{22}{15} = 1\frac{7}{15}$.

Multiplication and Division of Fractions

*To **multiply** two fractions, we simply multiply their numerators and then multiply their denominators.*

So $\frac{2}{5} \times \frac{1}{3} = \frac{2\times 1}{5\times 3} = \frac{2}{15}$ *and* $1\frac{3}{4} \times 2\frac{1}{5} = \frac{7}{4} \times \frac{11}{5} = \frac{77}{20} = 3\frac{17}{20}$

*To **divide** fractions, change the division into a multiplication by inverting the fraction you are dividing by.*

So $\frac{3}{8} \div \frac{1}{5} = \frac{3}{8} \times \frac{5}{1} = \frac{15}{8} = 1\frac{7}{8}$ *and* $5\frac{3}{5} \div \frac{8}{9} = \frac{28}{5} \times \frac{9}{8} = \frac{7\times 9}{5\times 2} = \frac{63}{10} = 6\frac{3}{10}$

(Invert $^1/_5$ and multiply) *(Invert $^8/_9$ and multiply; 4 is divided into both 28 and 8)*

Exercise 6

Find the value of: (a) $\frac{2}{3}+\frac{3}{4}=$ (b) $\frac{4}{7}+\frac{1}{3}=$ (c) $3\frac{1}{2}+5\frac{3}{4}=$

(d) $\frac{3}{4}-\frac{2}{5}=$ (e) $\frac{4}{9}-\frac{3}{10}=$ (f) $4\frac{2}{5}-2\frac{1}{10}=$ (g) $1\frac{2}{3}+\frac{1}{4}-\frac{1}{6}=$

Exercise 7

Evaluate the following:

(a) $\frac{2}{3}\times\frac{5}{12}=$

(b) $\frac{7}{9}\times\frac{3}{4}=$

(c) $2\frac{1}{4}\times 3\frac{1}{5}=$

(d) $\frac{3}{4}\div\frac{1}{2}=$

(e) $\frac{5}{6}\div\frac{3}{4}=$

(f) $5\frac{2}{5}\div\frac{4}{15}=$

(g) $\frac{3}{8}\times\frac{2}{5}\div 1\frac{1}{2}=$

DECIMALS

All real numbers can be written in decimal form, based on powers of 10. Each digit of the number has a place value corresponding to its position in the decimal, which indicates its power of 10. Zeros are used as place markers. The decimal point of a number separates the whole part of the number from the part of the number less than one.

4583**.**609 is the short hand way of writing: $4000 + 500 + 80 + 3 + \frac{6}{10}+\frac{0}{100}+\frac{9}{1000}$

A fraction may be converted to a decimal by dividing the numerator of the fraction by its denominator.

So $^3/_8$ can be written as $3 \div 8 = 0.375$. The 3 in this decimal represents $^3/_{10}$. The 7 represents $^7/_{100}$. The 5 represents $^5/_{1000}$. The zero means there is no whole part in $^3/_8$.

Exercise 8

Write as decimals :(a) $^5/_{10}$ (b) $^6/_{100}$ (c) $^5/_{100} + {}^3/_{1000}$ (d) $2000 + 5 + {}^8/_{100}$

Exercise 9

Arrange from smallest to largest : 3.8 , 3 , 3.67 , 3.08 , 3.001 ; and write each as a fraction.

Exercise 10

What is the value of 6 in the following? (a) 32614.8 (b) 29.0651 (c) 2600731

(d) 10.0046

Exercise 11

Use your calculator to write as decimals: (a) $^4/_5$ (b) $^5/_9$ (c) $^{11}/_3$ (d) $4^3/_7$

Some rules for working with decimals

Adding or subtracting *two decimals. Be sure to line up the decimal points and corresponding digits with the same position under each other.*

Multiplying by 10 *or 100 or 1000, etc. Shift the decimal point to the* ***right*** *by the number of zeros.*

So if multiplying by 100, move the decimal point 2 places to the right.

***Dividing by 10** or 100 or 1000, etc. Shift the decimal point to the **left** by the number of zeros.*

So if dividing by 1000, move the decimal point 3 places to the left.

***Multiplying two decimals**. The number of decimal places given in the original question is equal to the number of decimal places in the answer. So 0.03 x 0.2 = 0.006 . Here two decimal places (after the decimal point) in 0.03, plus one decimal place in 0.2, makes 3 decimal places in the answer.*

***Dividing two decimals**. Multiply the denominator and then the numerator by the same number.*

So $0.45 \div 0.3 = \frac{0.45}{0.3} = \frac{0.45}{0.3} \times \frac{10}{10} = \frac{4.5}{3} = 1.5$ *thus 0.45 ÷ 0.3 is the same as 4.5 ÷ 3 = 1.5*

Exercise 12

Find the following without using a calculator. Check your answers using a calculator.

(a) 10.48 + 8.16 = (b) 106.17 – 92.41 = (c) 11.2 – 0.8 + 3.67 = (d) 0.3 x 0.17 =

(e) 13.06 x 8 = (f) $(0.05)^2$ = (g) 2.6721 x 100 = (h) 3.8 ÷ 10 =

(i) 0.63 ÷ 0.07 = (j) 12.111 ÷ 1.1 = (k) $\frac{4.25 \times 8}{0.5}$ = (l) $\frac{10(3.42-0.09)}{0.03}$ =

PERCENTAGES

Percent, % , means 'for each 100', so a percentage is a fraction of 100.

For example, 23% is 23 parts of a whole split into 100 parts , i.e. 23% = $^{23}/_{100}$ *.*

So 100% = $^{100}/_{100}$ *= 1 , the whole number.*

Decimals and fractions are easy to change into percentages, by multiplying by 100%.

So 0.76 = 0.76 x 100% = 76% and 0.083 = 0.083 x 100% = 8.3%.

Also $\frac{2}{5} = \frac{2}{5} \times \frac{100\%}{1} = \frac{200}{5}\%$ *= 40% , and it is easily seen that* $^{1}/_{2}$ *= 50% and* $^{1}/_{10}$ *= 10%.*

Exercise 13 Complete the following tables:

Fraction	Decimal	Percentage
$^{4}/_{5}$		
$^{1}/_{3}$		
	0.25	
		64%

Fraction	Decimal	Percentage
$^{5}/_{8}$		
$^{3}/_{16}$		
	2.35	
		0.3%

ANSWERS TO EXERCISES
KINDS OF NUMBERS
Exercise 1
(a) 10 , 3, 7, 21, 37 (b) 10 , 0 , 3 , 7, 21 , 37

(c) 10 , 0 , 3 , 7, -2 , 21 , 37 (d) -9.001, $-\sqrt{7}$, $-1\frac{1}{4}$, $-\frac{3}{5}$, -2

(e) -9.001, 10 , $-1\frac{1}{4}$, 0 , 3 , 8.94 , $-\frac{3}{5}$, 7, -2 , 21 , 37

(f) $-\sqrt{7}$, $\sqrt{5}$ (g) 3 , 7 , 37 (h) 10 , 21

INEQUALITIES
Exercise 2
(a) T (b) T (c) T (d) T (e) F [= –8] (f) F [$\sqrt{97}$ < 10] (g) F [-2.01<-2] (h)T [5>3]

(i) F [24<$\sqrt{577}$] (j) -5<x<4 (k) x≥10 (l) -5≤X≤4 (m) x≤10 (n) -5<X≤4

FRACTIONS
Exercise 3. (a) $^{8}/_{10}$ (b) $^{15}/_{20}$ (c) $^{25}/_{75}$ (d) $^{30}/_{80}$

Exercise 4 (a) $^{3}/_{4}$ (b) $^{5}/_{8}$ (c) $^{7}/_{13}$ (d) $^{5}/_{7}$ (e) $^{2}/_{3}$

Exercise 5 (a) $^{5}/_{3}$ (b) $^{5}/_{2}$ (c) $^{65}/_{7}$ (d) $3^{2}/_{5}$ (e) $12^{1}/_{2}$ (f) $7^{1}/_{3}$

Exercise 6 (a) $^{17}/_{12} = 1^{5}/_{12}$ (b) $^{19}/_{21}$ (c) $8+ {^{5}/_{4}} = 9^{1}/_{4}$ (d) $^{7}/_{20}$ (e) $^{13}/_{90}$

(f) $^{22}/_{5} - {^{21}/_{10}} = {^{23}/_{10}} = 2^{3}/_{10}$ (g) $^{20}/_{12} + {^{3}/_{12}} - {^{2}/_{12}} = {^{21}/_{12}} = 1^{9}/_{12}$

Exercise 7 (a) $^{5}/_{18}$ (b) $^{7}/_{12}$ (c) $^{36}/_{5} = 7^{1}/_{5}$ (d) $^{3}/_{2}$ (e) $^{10}/_{9} = 1^{1}/_{9}$

(f) $^{81}/_{4} = 20^{1}/_{4}$ (g) $^{1}/_{10}$

DECIMALS
Exercise 8 (a) 0.5 (b) 0.06 (c) 0.053 (d) 2005.08

Exercise 9 (a) smallest to largest 3 , 3.001 , 3.08 , 3.67 , 3.8

(b) fractions $3.8 = 3\ {^{8}/_{10}}$, $3 = {^{3}/_{1}}$, $3.67 = 3 + {^{6}/_{10}} + {^{7}/_{100}} = 3\ {^{67}/_{100}}$, $3.08 = 3\ {^{8}/_{100}}$,

$3.001 = 3^{1}/_{1000}$

Exercise 10 (a) 600 (b) $^{6}/_{100}$ (c) 600000 (d) $^{6}/_{10000}$

Exercise 11 (a) 0.8 (b) 0.5555… (c) 3.6666… (d) 4.428571428571…

Exercise 12 (a) 18.64 (b) 13.76 (c) 14.07 (d) 0.051 (e) 104.48

(f) 0.0025 (g) 267.21 (h) 0.38 (i) 9 (j) 11.01

(k) 68 (l) 1110

PERCENTAGES
Exercise 13

Fraction	Decimal	Percentage
$^{4}/_{5}$	0.8	80%
$^{1}/_{3}$	0.333…	$33^{1}/_{3}$ %
$^{1}/_{4}$	0.25	25%
$^{64}/_{100}$	0.64	64%

Fraction	Decimal	Percentage
$^{5}/_{8}$	0.625	62.5%
$^{3}/_{16}$	0.1875	18.75%
$2\ {^{35}/_{100}}$	2.35	235%
$^{3}/_{1000}$	0.003	0.3%