

Module 2: Computation

ROUNDING

Often numbers are rounded as it is not useful or necessary to use the exact value. For example, calculating 76% of \$12.30, the exact answer is \$ 9.348. Clearly this is an impractical amount and so we round \$9.348 to the nearest cent, \$9.35. A decimal number may be rounded to any chosen place value. For example, 786.4, rounded to the nearest unit is 786, to the nearest tens is 790 and to the nearest hundred is 800

To round to a given place value, look at the first digit to the right of the given place value, and determine whether the **next** digit is 5 or more, if so, round upwards. If less than five, round downwards.

Worked Examples:

15.6705 to the nearest ten is 20

15.6705 to the nearest unit is 16

15.6705 to the nearest tenth is 15.7

15.6705 to the nearest hundredth is 15.67

15.6705 to the nearest thousandth is 15.671

Exercise 1

Round the following decimals to the specified place value.

1. 2.953 to the nearest hundredth =
2. 2.5 to the nearest unit =
3. 1.5389 to the nearest thousandth =
4. 6.396 to the nearest tenth =
5. 8.11985 to the nearest ten thousandth =
6. 1.987654 to the nearest hundred thousandth =
7. 3.75938 to the nearest thousandth =
8. 0.3125 to the nearest thousandth =

ORDER OF OPERATIONS

Mathematical operations must be done in the correct order. The order of operations follows:

Brackets	First calculate operations within brackets
Exponents	Second, calculate any expression with exponents. In other words, calculate the given power of any number.
Multiplication or Division	Third, multiply or divide. Working from left to right if there is more than one multiple or divide in the expression.
Addition or Subtraction	Fourth, add or subtract, working from left to right.

Worked examples:

$$\begin{aligned} \text{(a)} \quad & 8 + 12 \div 4 - 5 + 6 \times 2 \\ & = 8 + 3 - 5 + 6 \times 2 \quad \text{Divide} \\ & = 8 + 3 - 5 + 12 \quad \text{Multiply} \\ & = 11 - 5 + 12 \quad \text{Add} \\ & = 6 + 12 \quad \text{Subtract} \\ & = 18 \quad \text{Add} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & 2^3 + 3^2 - 13 \times 2 \\ & = 8 + 9 - 13 \times 2 \quad \text{Exponents} \\ & = 8 + 9 - 26 \quad \text{Multiply} \\ & = 17 - 26 \quad \text{Add} \\ & = -9 \quad \text{Subtract} \end{aligned}$$

(c) To simplify fractions with a combination of operations in the numerator and denominator, calculate the numerator and denominator separately first and then the division last.

$$\frac{6 + 7^2 - 56 / 8}{32 \times 2 / 8} = \frac{6 + 49 - 7}{64 / 8} = \frac{55 - 7}{8} = \frac{48}{8} = 6$$

Arithmetic operations with negative numbers

When **adding a negative number** move to the **left** of the number line.

$$\text{e.g. } 5 + (-2) = 3 \quad 5 + (-8) = -3 \quad -5 + (-2) = -7 \quad -5 + (-8) = -13$$

When **subtracting a negative number** move to the **right** of the number line.

$$\text{e.g. } 5 - (-2) = 7 \quad 5 - (-8) = 13 \quad -5 - (-2) = -3 \quad -5 - (-8) = 3$$

When **multiplying** or **dividing**, **like signs give a positive** result, **opposite signs give a negative** result.

$$\begin{aligned} \text{e.g. } \quad & (4)(6) = 4 \times 6 = 24 & (-3)(-5) = (-3) \times (-5) = 15 & (4)(-6) = 4 \times -6 = -24 \\ & (-3)(5) = (-3) \times 5 = -15 & 24/6 = 4 & -24/(-6) = 4 & -24/6 = -4 & 24/-6 = -4 \end{aligned}$$

Know your calculator: Check the answers given in the worked examples above. Find the 'negative' button on your calculator.

Some calculators add brackets when the negative button is pressed, while others will require you to add brackets.

Exercise 2

Evaluate the following, without a calculator. Check your answer with the calculator.

$$\text{(a)} \quad 5 + (-12) =$$

$$\text{(k)} \quad 121 - 4^2(2) =$$

$$\text{(t)} \quad \frac{2 + \sqrt{9^2 - (4)(2)(10)}}{4} =$$

$$\text{(b)} \quad -3 - (-9) =$$

$$\text{(l)} \quad \sqrt{49}(-3) / 7 =$$

$$\text{(c)} \quad -5 + (-6) =$$

$$\text{(m)} \quad 22 - (2)(4)(-5) - 12 =$$

$$\text{(d)} \quad -8 + 13 =$$

$$\text{(n)} \quad 100 - 30 / 10 \times 2 =$$

$$\text{(e)} \quad 5 - 9 =$$

$$\text{(o)} \quad 100 - 30 / (10 \times 2) =$$

$$\text{(f)} \quad -1 - 2 =$$

$$\text{(p)} \quad \sqrt{81} + 2^1 + 4^3 =$$

$$\text{(g)} \quad (7)(8) =$$

$$\text{(q)} \quad \frac{36-22}{7} =$$

$$\text{(h)} \quad (-9)(-9) =$$

$$\text{(r)} \quad \frac{(21-30)}{3/\sqrt{36}} =$$

$$\text{(i)} \quad -6 / 3 =$$

$$\text{(s)} \quad 5(3) + (8 - 5)^2 / 9 =$$

$$\text{(j)} \quad 49 / (-7) =$$

SUMMATION NOTATION, Σ (For statistic students)

The Greek letter sigma ' Σ ' means to 'find the sum of'.

Let say we have a set of ten numbers, $X = \{8, 3, 2, -9, 6, 0, -1, -4, 5, 2\}$, then the sum of X , written ΣX , is the sum of all the numbers in the set X . If we add up all the numbers in the set X , the answer is 12, and we write, $\Sigma X = 12$.

[Check with your calculator: $8 + 3 + 2 + (-9) + 6 + 0 + (-1) + (-4) + 5 + 2 = 12$].

To find ΣX^2 , the sum of the X squares, first square all of the numbers in the set X, and then add up the individual X^2 . The $\Sigma X^2 = 240$.

[Check with your calculator: $8^2 + 3^2 + 2^2 + (-9)^2 + 6^2 + 0^2 + (-1)^2 + (-4)^2 + 5^2 + 2^2 = 240$]

Here are some more worked examples using summation notation.

If the set $Y = \{8, 3, 2\}$

The sum of $(Y-3)$ is 4, or $\Sigma(Y-3)=4$. *First* subtract 3 from each Y, then add.

$$\Sigma(Y-3) = (8-3) + (3-3) + (2-3) = 5 + 0 + -1 = 4$$

The sum of $(Y-3)^2$ is 26, or $\Sigma(Y-3)^2 = 26$. *First* subtract 3 from each number, second square each of these numbers and third add.

$$\Sigma(Y-3)^2 = (8-3)^2 + (3-3)^2 + (2-3)^2 = 25 + 0 + 1 = 26$$

Remember, the summation (adding up) is done last.

Exercise 3

Below, two data sets (a set of numbers called X and another set of numbers called Y) have been entered into the first two columns. Calculate the missing summations given in the **last** row of the table.

X	Y	X^2	X-2	$(X-2)^2$	XY
8	0	$8 \times 8 = 64$	$8 - 2 = 6$	$(8 - 2)^2 = 6^2 = 36$	$(8)(0) = 0$
3	5	$3 \times 3 = 9$			
2	4			$(2 - 2)^2 = (0)^2 = 0$	
-9	8	81		$(-9 - 2)^2 = (-11)^2 = 121$	
6	4		$6 - 2 = 4$		
0	8	0			
-1	6	1			$(-1)(6) = -6$
-4	4	16	$-4 - 2 = -6$		
5	0				
2	4	4			$(2)(4) = 8$
$\Sigma X =$	$\Sigma Y = 43$	$\Sigma X^2 = 240$	$\Sigma(X-2) =$	$\Sigma(X-2)^2 =$	$\Sigma XY =$

PERCENTAGES

Worked examples:

1. Find 14% of \$325. Remember 'of' means multiply, so we calculate $\frac{14}{100} \times \frac{\$325}{1} = \$45.50$.
2. Increase 85 kg by 20%. To increase by 20%, we have the original 100% plus the new 20%, which is 120% of 85 kg in total. So the increased value will be $\frac{120}{100} \times \frac{85}{1} = 102$ kg.
3. Decrease \$35 by 12%. To decrease by 12%, we have the original 100% subtract the new 12%, which is 88% of \$35. So the decreased value will be $\frac{88}{100} \times \frac{35}{1} = \30.80
4. Write 240 as a percentage of 320. This is just a fraction to be written as a percentage.
So we look at $\frac{240}{320} = \frac{3}{4}$ and then change to % $\frac{3}{4} = \frac{3}{4} \times \frac{100\%}{1} = 75\%$
(or using a calculator $240 \div 320 \times 100 = 75\%$)
5. Find the percentage decrease when \$5 400 falls to \$4725. A percentage change is
Here the change is a decrease of \$5 400 - \$4 725 = \$675.

$\frac{\text{change}}{\text{original}} \times 100\%$
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 So the % decrease is $\frac{675}{5400} \times 100\% = 12.5\%$.

Exercise 4

1. Determine (a) 12% of 300 (b) 45% of 630
(c) 11.8 % of \$15 360 (d) a 2% increase on 720
(e) a price of \$3500 discounted by 15% (f) 3.8% more than 54.9 km
2. Find (a) 7 kg as a percentage of 10 kg (b) 45 as a % of 54
(c) 85 cents as a % of \$4
and the percentage change when
(d) \$63 grows by \$21 (e) 4.5 L drops to 3.6 L
(f) 3.5 km stretches to 5.6 km
3. At the laboratory the analyst finds that for the ore samples analysed 345 out of 923 contained traces of gold. Find percentage of ore samples that contained traces of gold. (Round the percentage to nearest tenth)
4. A population of black beetles on Emu Island grew by 4% over the year 2014 due to favourable climatic conditions. At the end of 2013 there was a population of 14 million beetles.
(a) How many were there at the end of 2014?
(b) If the population grows at the same rate in 2015, how many beetles will there be at the end of 2015?
5. In 2012 the population of a species of monkeys in Borneo decreased by 2.15%. At the beginning of the year there were 8000 monkeys. What was the population at the end of the year?
6. The %error in a measurement is given by the formula .

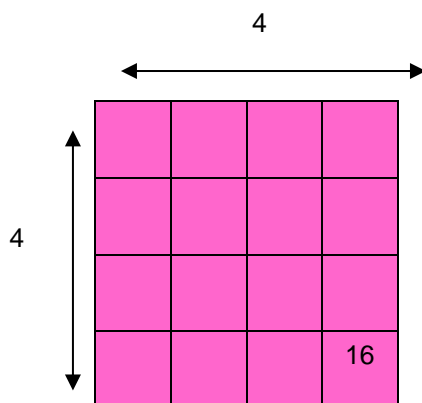
$\frac{ \text{measured value} - \text{actual value} }{\text{actual value}} \times 100\%$
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A student estimated the number of jelly babies in a jar was 126. However, the actual number in the jar was 150. Calculate the % error of the estimate.

DECIMALS

Exercise 5

1. The minimum temperature in Perth has dropped in the last few days. On Monday the temperature was 11.6°C , Tuesday it dropped to 5.8°C and on Wednesday it was 4.8°C . What was the average minimum temperature over the three days? (Round to the nearest tenth)
2. On a holiday to the south-west, you drove $465\frac{6}{10}$ kms using $44\frac{1}{10}$ litres of petrol. What is the fuel economy of the car for this trip, in kilometres per litre? (Round to the nearest tenth)
3. Packets of jelly babies are filled by weight. If I buy a 1300 gram packet of jelly babies and I know that 5 jelly baby weighs 25 grams, how many jelly babies are in the packet?
4. A rectangular pool 5.3m by 8.2m is to be covered by a solar pool blanket. What is the total cost of the blanket at \$7.25 per square metre?



POWERS

Worked examples:

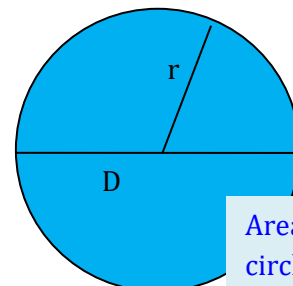
For a square with sides of 4cm in length, the area of the square is $4^2 = 16 \text{ cm}^2$.

If the area of a square is 121cm^2 , then the length of the sides will be $\sqrt{121} = 11\text{cm}$.

A circle with a radius of 3cm has an area of $\pi \times 3^2 =$

$3.141593 \times 3^2 = 28.3 \text{ cm}^2$ (rounded to the nearest tenth), the area of a circle = πr^2 .

If the area of a circle is 1257mm^2 , then the radius of the circle will be $\sqrt{\text{Area}/\pi} = \sqrt{1257/\pi} = 20\text{mm}$.



Area of a circle = πr^2

Exercise 6

1. The diameter of a 20 cent piece is 30mm and the diameter of a 5 cent piece is 20mm. How much larger is the area of a 20 cent piece?
2. A square room has a floor area of 23 square metres. What is the length of the each side of the room, to the nearest tenth of a metre?
3. The area within an ancient stone circle covers 491m^2 . What is the diameter of the stone circle to the nearest metre?

ANSWERS

Rounding

Exercise 1

(1.) 2.95 (2.) 3 (3.) 1.539 (4.) 6.4 (5.) 8.1199 (6.) 1.98765 (7.) 3.759 (8.) 0.313

Order Of Operations

Exercise 2

(a) -7 (b) 6 (c) -11 (d) 5

(e) -4 (f) -3 (g) 56 (h) 81

(i) -2 (j) -7

(k) $121 - 4^2(2) = 89$

(l) $\sqrt{49}(-3) \div 7 = -3$

(m) $22 - 2(4)(-5) - 12 = 50$

(n) $100 - 30 \div 10 \times 2 = 94$

(o) $100 - 30 \div (10 \times 2) = 98.5$

(p) $\sqrt{81} + 2^1 + 4^3 = 75$

(q) $\frac{36-22}{7} = 2$

(r) $(21 - 30) \div (3 \div \sqrt{36}) = -18$

(s) $5(3) + (8 - 5)^2 / 9 = 16$

(t) $\frac{2 + \sqrt{9^2 - (4)(2)(10)}}{4} = 0.75$

Summation Notation

Exercise 3

X	Y	X ²	X-2	(X-2) ²	XY
8	0	64	6	36	0
3	5	9	1	1	15
2	4	4	0	0	8
-9	8	81	-11	121	-72
6	4	36	4	16	24
0	8	0	-2	4	0
-1	6	1	-3	9	-6
-4	4	16	-6	36	-16
5	0	25	3	9	0
2	4	4	0	0	8
12	43	240	-8	232	-39

Percentages

Exercise 4

1. (a) 36 (b) 283.5 (c) \$1812.48 (d) 734.4 (e) \$2 975 (f) 56.9862 km

2. (a) 70% (b) 83¹/₃ % (c) 21.25% (d) 33¹/₃ % increase (e) 20% decrease

(f) 60% increase

3. 37.4% 4. (a) 14,560,000 (b) 15,142,400

5. 7 828

6. 16%

Decimals

Exercise 5

1. 7.4°C

2. 10.6 kms/litre

3. 260

4. $5.3 \times 8.2 \times \$7.25 = \315.085 or \$315

Powers

Exercise 6

1. 2.25 times larger

2. 4.8m

3. Diameter = 25m